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ESTIMATION OF SHEAR COMPLIANCE OF CONNECTORS IN COMPOSITE TWO-LAYER BEAMS BASED ON THE ANALYSIS OF FREE VIBRATIONS

Abstract

The article shows possibilities in a diagnostic of composite two-layer beams with flexible connectors (like bolts in steel-concrete beams and nails in wooden-concrete ones) offered by an analysis of their natural frequencies. Finding the minimum of error function (between model and measured natural frequencies of beam) enables an estimation of shear compliance for the interlayer joint of beam and, if needed, Young' modulus for one of layers.

Keywords

combined beam, shear stiffness, shear compliance, natural frequency, free vibrations, connector, stud, compliant joint, diagnostics

1 INTRODUCTION

Composite structures with compliant shear connections of their sub-elements become more and more popular systems in the building industry. The first of all one should appreciate for this type of structures a considerable increase of their load capacity and stiffness resulting from combining their components. For example in case of composite steel-ferroconcrete (or wood-ferroconcrete) beams an upper ferroconcrete slab is mainly compressed and the lower bar steel (or wooden) element is subject to tension, what is extremely advantageous in view of the properties of used materials. Moreover the problem of torsional-flexural buckling of more flexible steel (or wooden) profiles is eliminated thanks to connecting them with the rigid upper slab [3]. It should be noticed that these kinds of beam structures work correctly if the shear contact between the ferroconcrete slab and steel (wooden) bar elements shows enough low compliance. However this property may increase during a long-lasting exploitation in effect of corrosion of the stud (bolt) connectors and concrete in the joint, evolution of fatigue micro-cracks in the concrete slab etc. Therefore a way of estimating the shear compliance for the beam interlayer joints based on an analysis of their natural frequencies is proposed in the paper. Such a kind of diagnostic tests can be carried out on real structures in a non-invasive way. The considerations are finally illustrated by a computational example for a steel-ferroconcrete combined beam.

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2 MATHEMATICAL DESCRIPTION OF THE PROBELM

Let us consider a composite beam which consists of two elastic layers of different Young's modulus (E_1 and E_2) and bisymmetric cross-sections. A contact between these two elements is assumed as a flexible one of shear compliance k^{-1}. For example a static scheme for such a free-ends beam and its differential element are presented in Fig.1. It is subject to an action of distributed, external and time-variable load q.

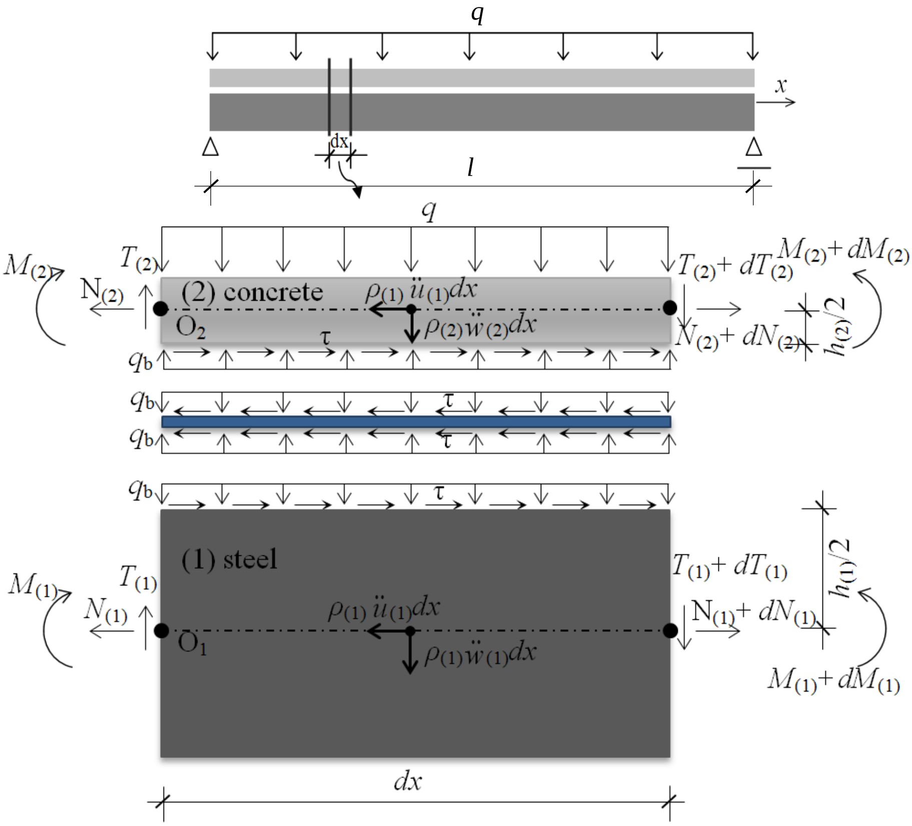

Fig.1: The static scheme of steel-ferroconcrete beam and its differential element with the forces acting over it

The symbols used in Fig.1, that is: $M_{(i)},T_{(i)},N_{(i)},u_{(i)},h_{(i)},O_{(i)},\rho_{(i)},q_b,\tau,w$, have the following meanings respectively: bending moment, shearing and axial force, horizontal displacement, height, position of cross-sectional mass centre and material density for the beam layer (i) (i=1,2), part of vertical load q taken over by the layer (1), shearing load in the contact of layers and vertical displacement of the axes of layers. The flexibility of contact enables a slip in the plane of connection. As shown in Fig.2 the slip s can be calculated from the equation [4]:

$$s = u_{(1)} - u_{(2)} - ew^{I}\,, \tag{1}$$

where: $e=½(h_{(1)}+h_{(2)})$, $(\dots)^I$ – the first derivative of function over x-axis, w^I – angle of rotation of centre line of the beam. Thus the shearing load in the elastic contact may be described by the formula:

$$\tau = ks = k(u_{(1)} - u_{(2)} - ew^{I})\,, \tag{2}$$

where: k – stiffness of the connection.

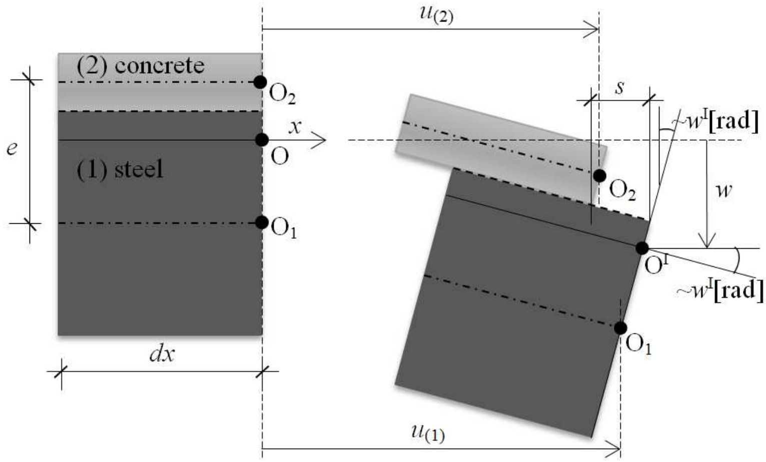

Fig.2: The slip in the contact of beam layers

Taking into account the physical equation (2) and the equilibrium conditions for the beam differential element shown in Fig.1 the following system of differential equations on functions of displacements w, $u_{(1)}$ and $u_{(2)}$ may be obtained:

$$\begin{cases} w^{IV}(E_{(1)}I_{(1)}+E_{(2)}I_{(2)})-ke^2w^{II}-ke^2w^{I} + keu_{(1)}^{I}+k^{I}eu_{(1)} - keu_{(2)}^{I}-k^{I}eu_{(2)} = -q-\rho\ddot{w} \\ kew^{I} + E_{(1)}A_{(1)}u_{(1)}^{II}-ku_{(1)} + ku_{(2)} = \rho_{(1)}\ddot{u}_{(1)} \\ -kew^{I} + ku_{(1)} + E_{(2)}A_{(2)}u_{(2)}^{II}-ku_{(2)} = \rho_{(2)}\ddot{u}_{(2)} \end{cases} \quad (3)$$

where: $(\ldots)^{II}$, $(\ldots)^{IV}$ – the second and fourth derivatives of a function over x-axis; $(\ddot{\ldots})$ – the second time-derivative; $I_{(i)}$- moments of inertia for the layer (i) (i=1,2) in the cross-section of beam layer. In order to solve the system it has to be completed by boundary and initial conditions. For example they can be formulated for the free-ends beam shown in Fig.1 as follows:

$$w(x=0,t)=0,\ w(x=l,t)=0,\ w^{II}(x=0,t)=0,\ w^{II}(x=l,t)=0,\ u_{(1)}^{I}(x=0,t)=0,$$
$$u_{(1)}^{I}(x=l,t)=0,\ u_{(2)}^{I}(x=0,t)=0,\ u_{(2)}^{I}(x=l,t)=0,\ w(x,t=0)=\tilde{w},\ \dot{w}(x,t=0)=\tilde{v}, \quad (4)$$
$$u_{(1)}(x,t=0)=\tilde{u}_{(1)},\ \dot{u}_{(1)}(x,t=0)=\tilde{v}_{(1)},\ u_{(2)}(x,t=0)=\tilde{u}_{(2)},\ \dot{u}_{(2)}(x,t=0)=\tilde{v}_{(2)},$$

where: t – time; $\tilde{w}$, $\tilde{v}$, $\tilde{u}_{(1)}$, $\tilde{v}_{(1)}$, $\tilde{u}_{(2)}$, $\tilde{v}_{(2)}$ – known functions.

Using Finite Element Method (FEM) or Finite Difference Method (FDM) we can express the system (3) generally in the matrix form:

$$\mathbf{Ku}=\mathbf{P}-\mathbf{B\ddot{u}}, \quad (5)$$

where: $\mathbf{K}$ – stiffness matrix, $\mathbf{u}$ – vector of node displacements, $\mathbf{P}$ – vector of forces (three terms in the equations), $\mathbf{B}$ – inertia matrix. In order to carry out the modal analysis of the problem we assume in the equation (5) $\mathbf{P}$=$\mathbf{0}$ and $\mathbf{u}=\mathbf{u}_0\sin(\omega t+\varphi)$ (e.g. [1]). Then we can obtain the following equation for eigenvalues λ_i of matrix $\mathbf{KB}^{-1}$:

$$\left(\mathbf{K}-\omega^2\mathbf{BI}\right)\mathbf{u}_0\, sin(\omega t+\varphi)=\mathbf{0} \rightarrow det\left(\mathbf{KB}^{-1}-\lambda\mathbf{I}\right)=0 \rightarrow \lambda_i=\omega_i^2, \quad (6)$$

where : $\mathbf{I}$ – unit matrix, $\mathbf{0}$ – zero vector, $\mathbf{u}_0$ – vector of free vibration amplitudes, ω_i – i-natural frequency of the beam, t – time.

Solving the above eigenvalue problem it is possible easily to calculate natural frequencies for the combined beam. On the other hand when natural frequencies of real combined beam are known from measurements (from the Fourier analysis of accelerations of chosen points in the real structure excited to test vibrations) it is possible to estimate its stiffness k finding the minimum of the following "error functions":

$$F=\sum_{i=1}^{n}\left|\frac{\omega_{i(\mathrm{measurement})}-\omega_{i(\mathrm{model})}}{\omega_{i(\mathrm{measurement})}}\right| \quad \text{or} \quad F=\sum_{i=1}^{n}\left(\frac{\omega_{i(\mathrm{measurement})}-\omega_{i(\mathrm{model})}}{\omega_{i(\mathrm{measurement})}}\right)^{2}, \tag{7}$$

where: $\omega_{i(\mathrm{measurement})}$– measured i-natural frequency for the real structure, $\omega_{i(\mathrm{model})}$– i-natural frequency calculated basing on the assumed model, n – number of the first natural frequencies taken into considerations. The function F depends on model stiffness k ($F=F(k)$) if $E_{(1)}$ and $E_{(2)}$ are known. However it is worth to notice in case of steel-ferroconcrete beams that Young's modulus of concrete is much more random than Young's modulus of steel because of the technology of fabrication. That is why it is sensible to consider in such a situation the error function as dependent on two variables, thus $F=F(E_{(\mathrm{concrete})},k)$. To illustrate possibilities of employing the above approach in diagnostics of combined beam structures the next sections are devoted to a presentation of computational example for a steel-ferroconcrete rib of floor in which upper ferroconcrete slab and steel I-profile are connected by Nelson bolts. The calculations were made by means of own computer program written in the MATLAB environment employing the above mathematical formulas and using FDM, what enabled to introduce the inertia matrix as a diagonal one.

3 DATA FOR THE COMPUTATIONAL EXAMPLE

Let us consider the example of composite steel-ferroconcrete floor with flexible interlayer shear connection created by steel studs (Fig.3) where the rib of floor can be treated as a free-ends beam from mechanical point of view to simplify the calculations. It is a kind of structure used mostly in the industry and bridge engineering. It enables to obtain relatively big span together with an advisable load-capacity and economic use of steel and concrete.

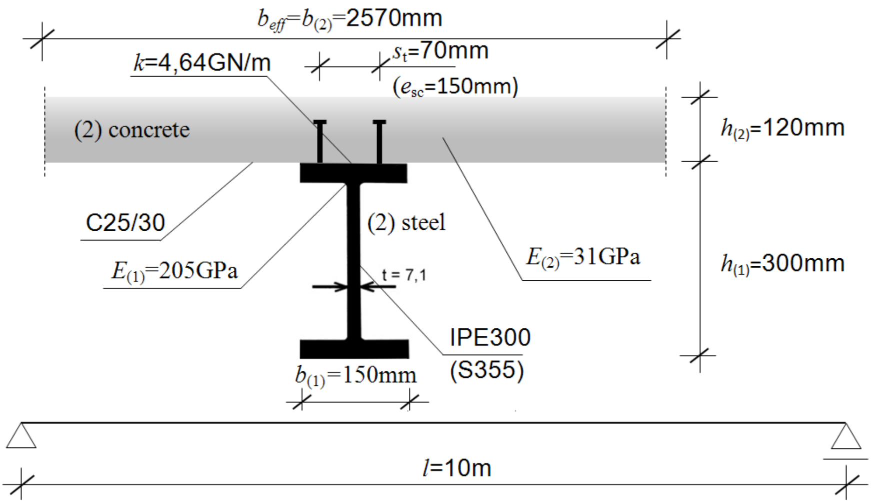

Fig.3: Cross-section and static scheme of the rib of analysed composite steel-ferroconcrete floor

The geometrical and material parameters assumed for the example are shown in Fig.3. The stiffness of stud connection (k=4,64GN/m) was established basing on the laboratory tests presented in the publication [2]. Finally one obtained for the beam three first values of natural frequencies as follows:

$$\omega_1 = 4.74\text{Hz}, \omega_2 = 18.64\text{Hz}, \omega_3 = 40.82\text{Hz}. \qquad (8)$$

4 ESTIMATION OF THE BEAM PARAMETERS

Because of the fact that Young's modulus of steel is much more predictable as mentioned in the section 2 it is purposeful to make the error function (7) dependent on the parameters $E_{(2)}$ and k for the example defined in the section 3. It is also worth to mention here that steel elements characterise most stable parameters in this kind of structures and regardless of types of steel used its modulus of elasticity is almost constant and equal to about 205GPa. The connectors are also made from steel but keeping the conditions concerning their shaping their load capacity and shear stiffness are determined the first of all by properties of concrete [2]. In view of the above considerations to present the possible diagnostic application of simple analysis of variation for the error functions (7) the contour line diagram of function (7_1) (for n=3) is presented in the Fig.4 for the example beam. The frequencies (8) are assumed here as $\omega_{(\text{measurement})}$. On the other hand the frequencies $\omega_{(\text{model})}$ are variable in dependence on $E_{(2)} \in [5,50]$GPa and $k \in [0.5,10]$GN/m. To visualise better the position of the global minimum the square root of this function is shown. It can be noticed that minimum in the diagram is unique in the ranges of $E_{(2)}$ and k which were chosen in order to analyse the physically acceptable conditions for the considered example because of the material limitations. It is also evident that if we only know enough exactly the geometry of structure, modulus of elasticity for one beam layer and as much as possible first natural frequencies (usually it is possible to measure from two to four of the first ones) then we can estimate sensibly the rest of essential mechanical properties for it.

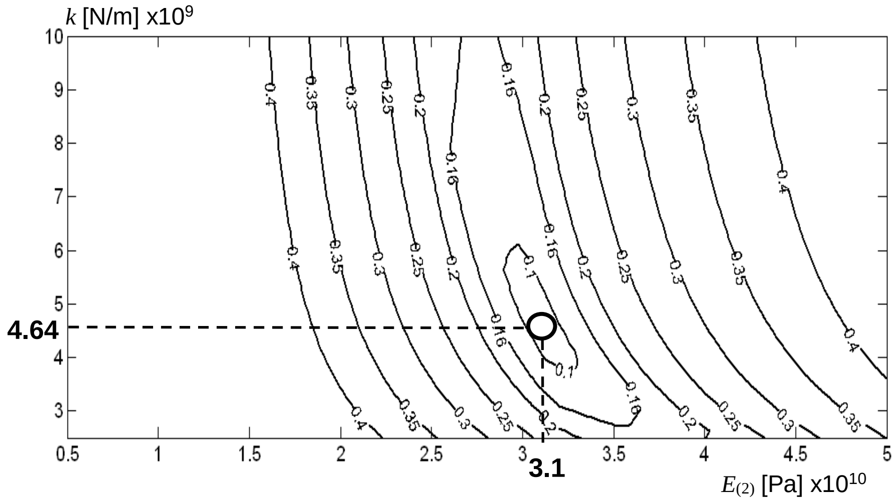

Fig.4: The contour line diagram of square root of error function (7_1) in the dependence on shear stiffness k and Young' modulus of concrete plate $E_{(2)}$ for the beam defined in the section 3

The task may be simplified considerably if both of layer moduli of elasticity are known. Then the analysis of variation of the error function can be made for the sake of one variable that is shear stiffness (compliance) of the interlayer connection. The diagram of square root of error function (7_1) is shown in the Fig.5 in this case for data from the section 3. One used in the presented example the method of systematic domain searching and the time needed for calculations amounted to about 20 minutes on the standard PC computer.

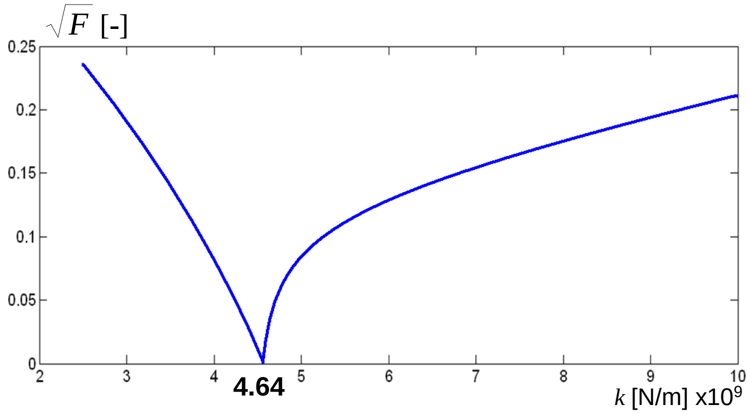

Fig.5: The diagram of square root of error function (7_1) in the dependence on shear stiffness k for the beam defined in the section 3

4 CONCLUSIONS

The combined structures (especially two-layer beams) are more and more popular and readily used because of their optimal use of materials with keeping advisable stiffness and load capacity. That is why non-destructive test methods should be intensively developed in this range too. The proposed method is based on the analysis of free vibrations. If we only can measure first two, three (or more if possible) natural frequencies it is possible to estimate other essential mechanical properties of combined beams, that is very important shear compliance (or stiffness) of interlayer connection (realised by studs, bolts, nails etc.) and more over Young's modulus for one of the layers. Because of the way of fabrication it is purposeful to estimate this way effective Young's modulus for concrete upper slab in steel-ferroconcrete beams what can be also used for an indirect estimation of its compression strength (e.g. basing on the formulas from the publication [6]). Of course a choice of way of searching the global minimum of error function and time-consumption of calculations rebated to this are other questions of the problem . That is why it is worth to choose more time-effective optimisation method than this used in the work (based for example on evolutionary algorithms (e.g. [5]) for more complex examples.

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